Visual Odometry FEB 9, 2021

1. Bayes Net for VO
2. Measurement Model
3. Noise Model
4. Nonlinear to Linear ~ GTSAM
5. Kalman filter as Least Squares
6. Normal Equations $H^T H \Delta x = H^T z$
7. Iter. Extended Kalman Filter, summary

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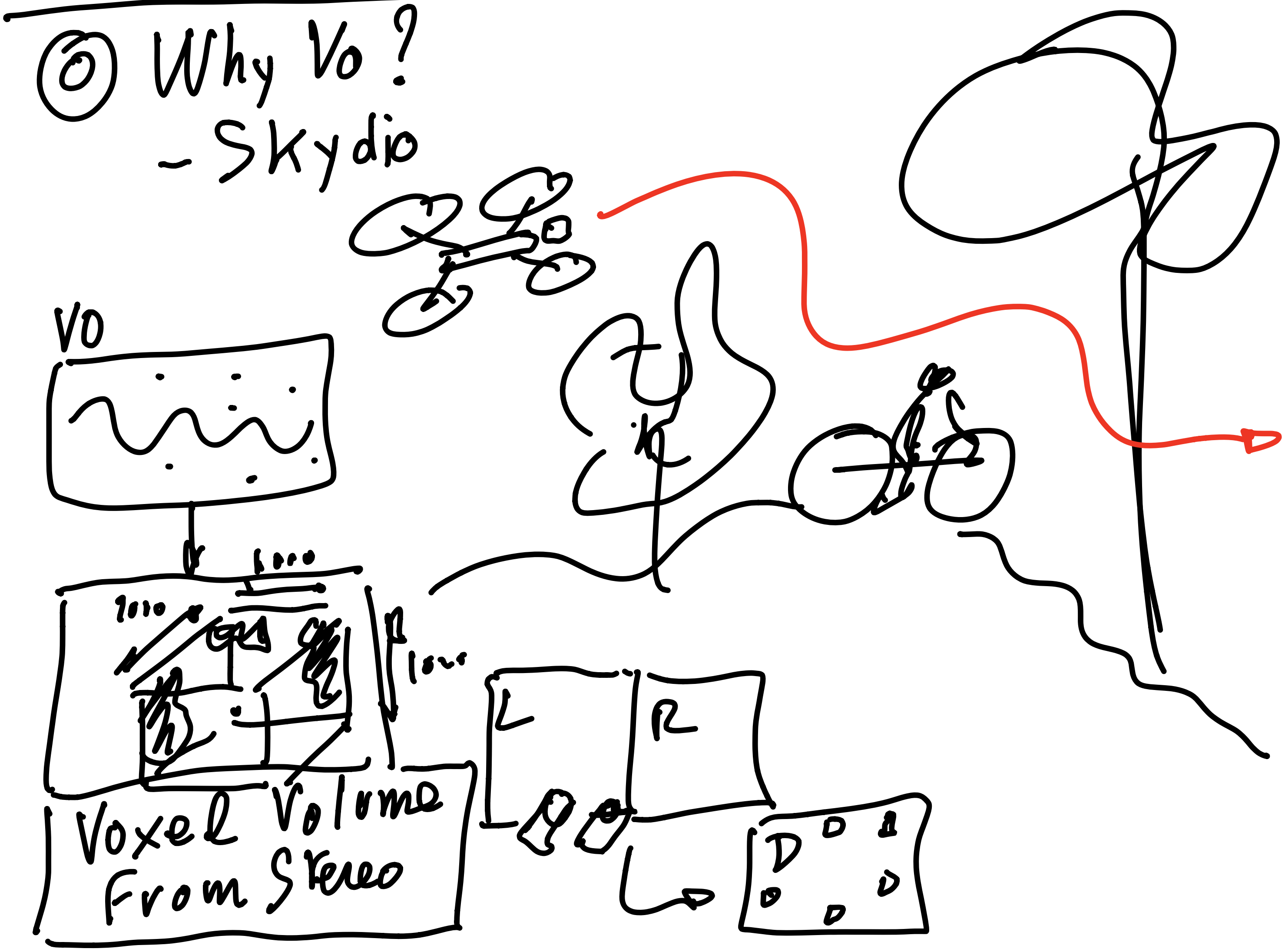
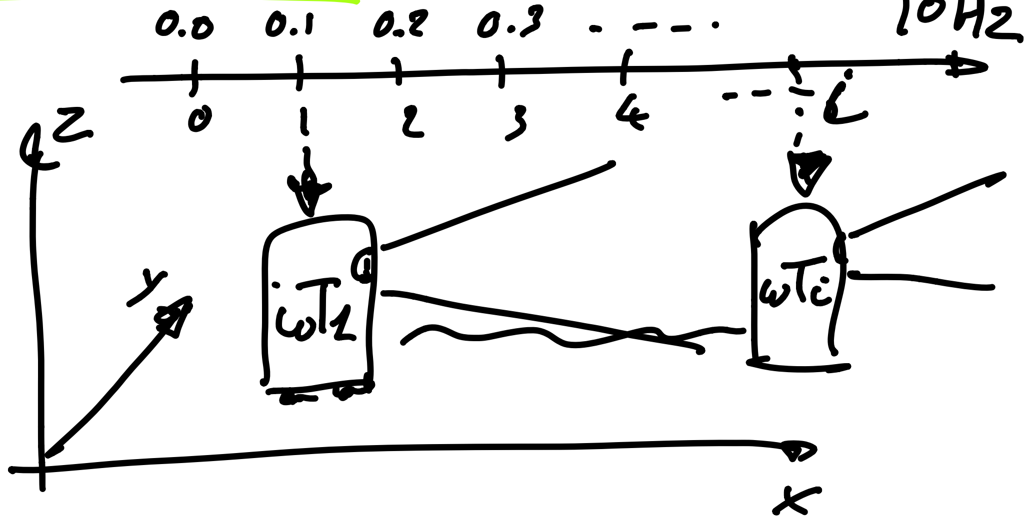
0. Why VO?
   - Skydio



① Bayes Net for VO



$_wT_i \doteq {}_wT_{B,t_i}$

$_wT_B$ possibilities:

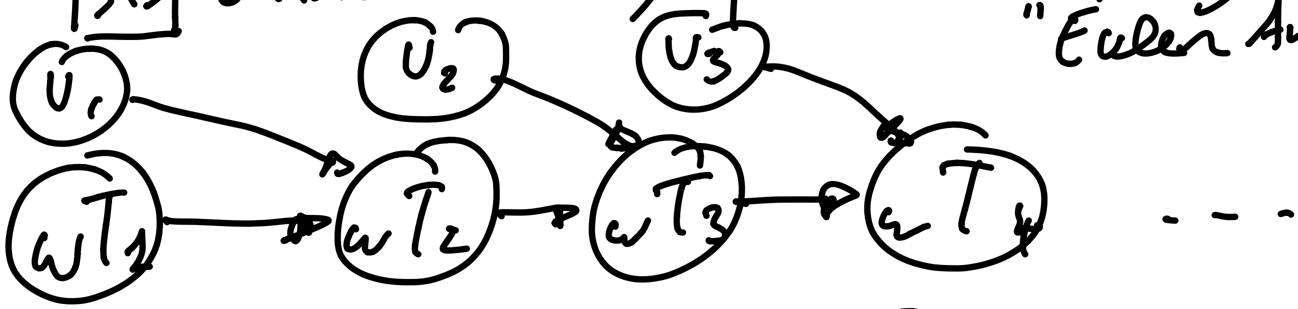
$(x, y) \in \mathbb{R}^2$

$(x, y, \theta) \in SE(2)$

$\underbrace{(x, y, z)} \in \mathbb{R}^3$

$(R, t) \in SE(3)$ "Special Euclidean group in 3D"

↳ $\boxed{3\times3}$ orthonormal R, $q \in Q$, (ψ, θ, φ) "Euler Angles"

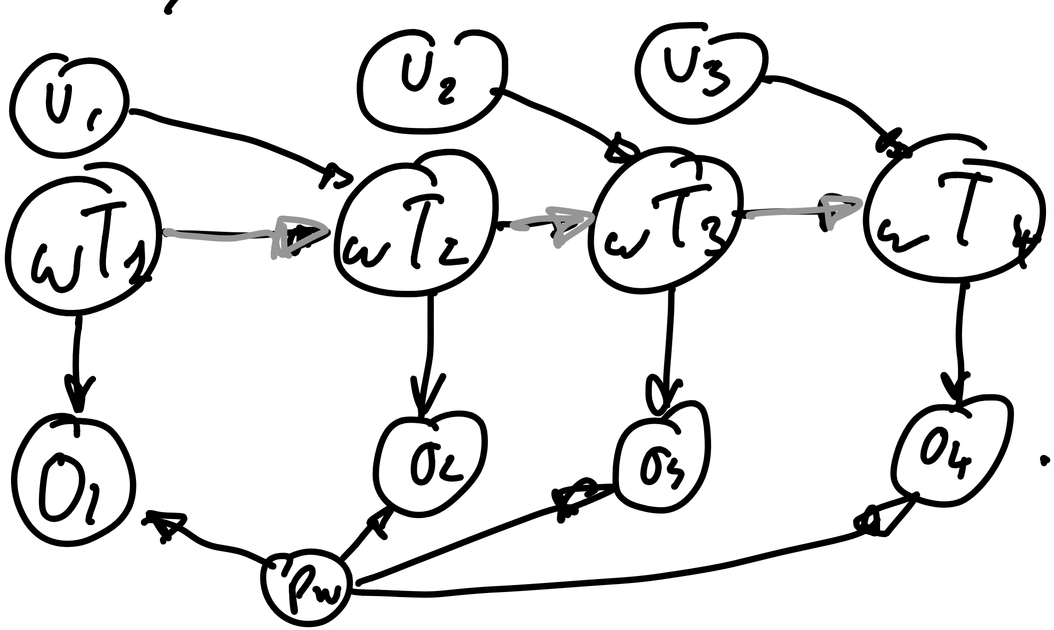


Mobile Robot:

$u \in \mathbb{R}^2$, $_wT_B \in SE(2)$

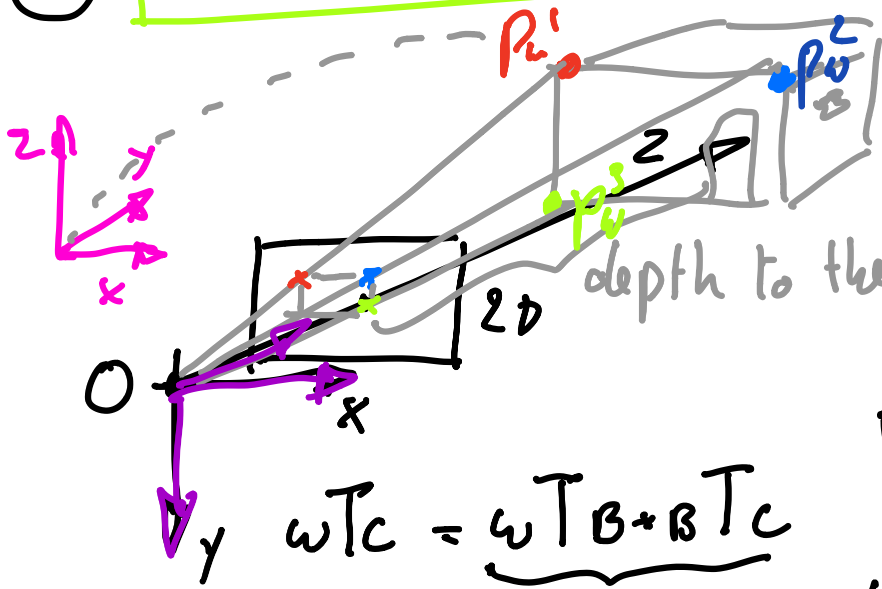
Drone:

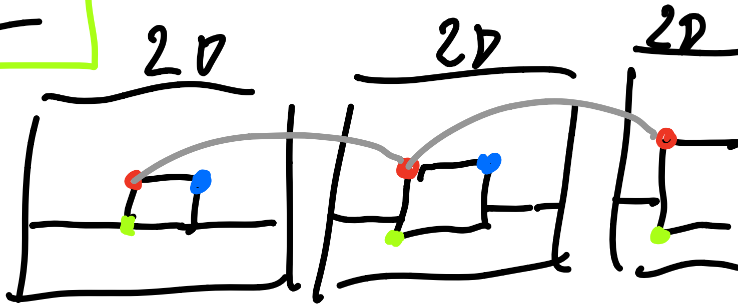
$u \in \mathbb{R}^4$, $_wT_B \in SE(3)$



- Simulate
  or
- {Bayes Filter

② Measurement Model





depth to the points Z

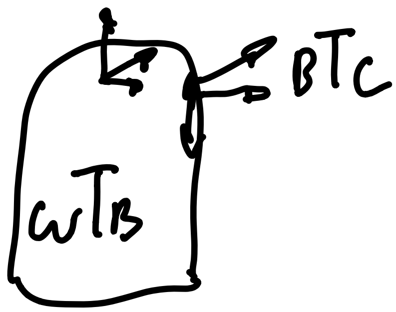
$${}^wT_c = \underbrace{{}^wT_B * {}^BT_c}$$

$${}^wT_c = {}^wT_B \cdot {}^BT_c$$

$$\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

matrix multiplication.

$$\underbrace{(R_1 R_2 \, , \, R_1 t_1 + t_2)}_{SE(2) \quad SE(3)}$$



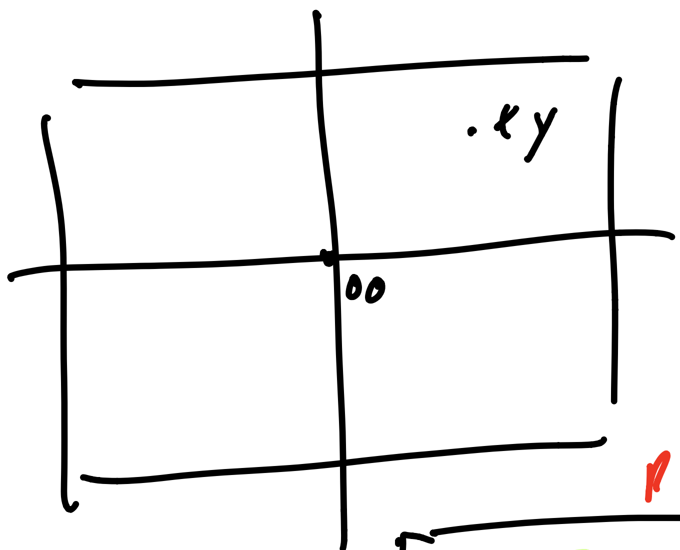
3D → 2D :

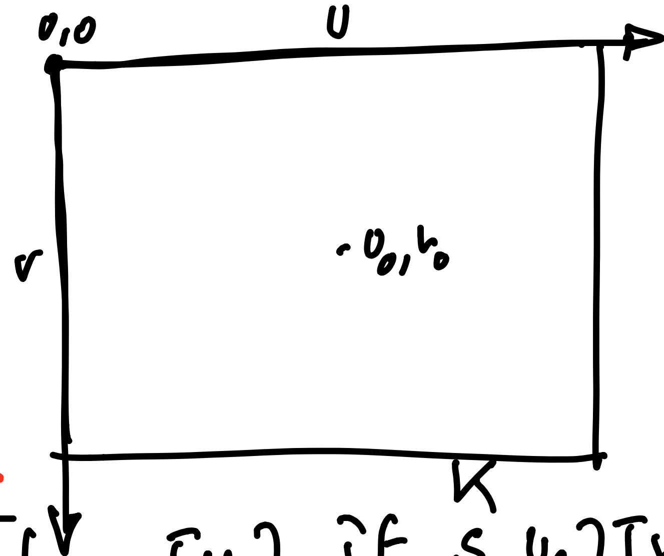
$$\boxed{\begin{cases} x = \dfrac{X}{Z} \\ y = \dfrac{Y}{Z} \end{cases}}$$

projection with known calibration in a perspective camera.

$\boxed{X \; Y \; Z}$ $\quad x, y$

Camera Frame!





Pixels

$$\begin{aligned} u &= u_0 + f x \\ v &= v_0 + f y \end{aligned}$$

K

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & u_0 \\ & f_y & v_0 \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$(u,v) = K(x,y)$$

$${}_wT_c = \underbrace{{}_wT_B \cdot {}_BT_c}$$

$$= K(\pi(XYZ))$$

$$= K(\pi(p_c))$$

$$= K(\pi(\underline{{}_cT_w} \cdot p_w))$$

$$= K(\pi({}_wT_c^{-1}\, p_w))$$

$$= \underline{K}(\underline{\pi}(\underline{({}_wT_B \cdot {}_BT_c)^{-1}}\ \underline{p_w}))$$

$$= h({}_wT_B, p_w ; {}_BT_c, K)$$

Full Sparse Measurement Model for VO

$$\boxed{= h({}_wT_B, p_w)}$$

+ noise

= generative model

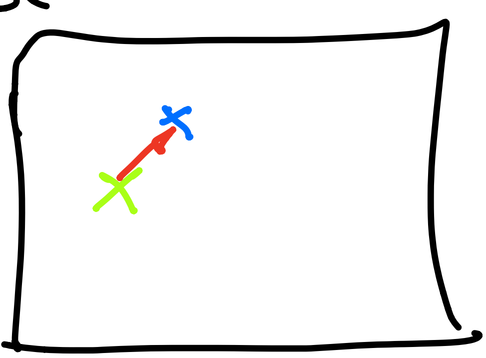
③ Noise Model

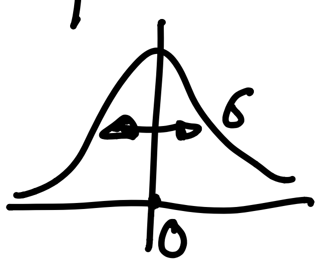
$$\underline{z_{ij}} = \underline{h({}_wT_i, p_w^j)} + \underline{\eta}$$

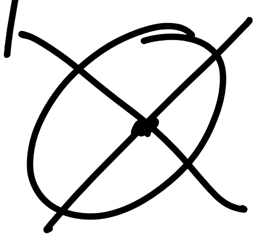
$\in \mathbb{R}^2$

$\eta \in \mathbb{R}^2$ $\quad \eta \sim \mathcal{N}\left(0, \begin{bmatrix} \sigma^2 & \\ & \sigma^2 \end{bmatrix}\right)$

2D







④ Nonlinear to Linear

$$f(x_0+\Delta x) = f(x_0) + f'(x_0)\Delta x + \frac{1}{2} f''(x_0)\Delta x^2 + \dots$$

$$\underbrace{z_{ij}}_{\in \mathbb{R}^2} = h({}_wT_i, p_w^j) + \eta$$

$$= \underbrace{h_0({}_wT_i^0, p_w^{j0})}_{\mathbb{R}^2} + \underset{2\times 6}{H}\,\underset{6}{\Delta T} + \underset{2\times 3}{G}\,\underset{3}{\Delta p} + \dots$$

Generalization of Taylor rule to

$$h: \mathbb{R}^6 \times \mathbb{R}^3 \to \mathbb{R}^2$$

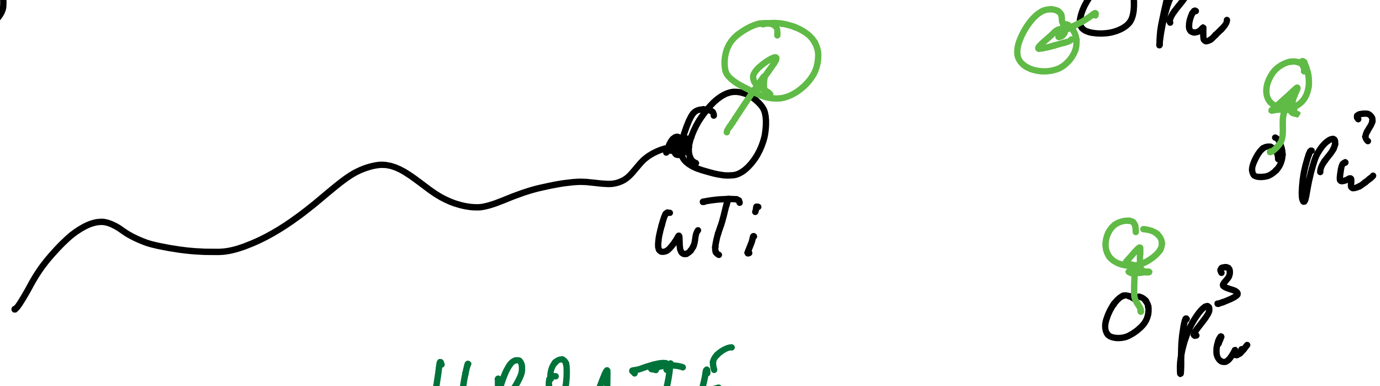
⑤ Kalman filter as Least Squares

Suppose we know ${}_wT_i^0$, p_w^{j0} (for 3 points)

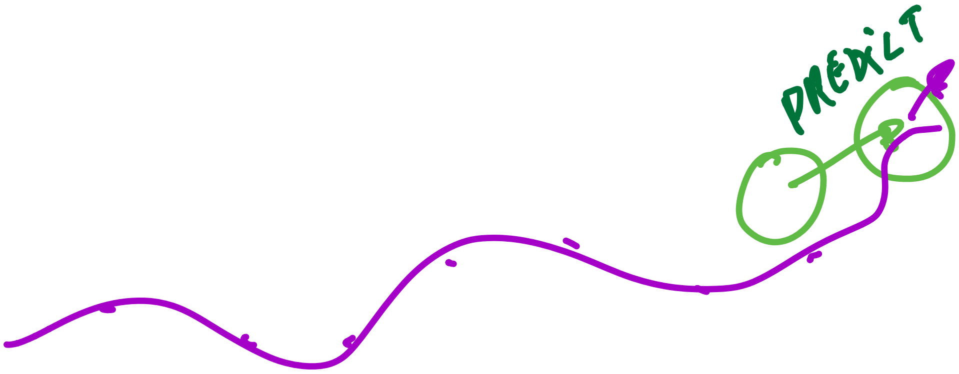
$$\forall_{ij} \quad h_0({}_wT_i^0, p_w^{j0}) + H\underset{6D}{\Delta T} + G\underset{3D}{\Delta p} \approx \underset{2D}{z_{ij}}$$

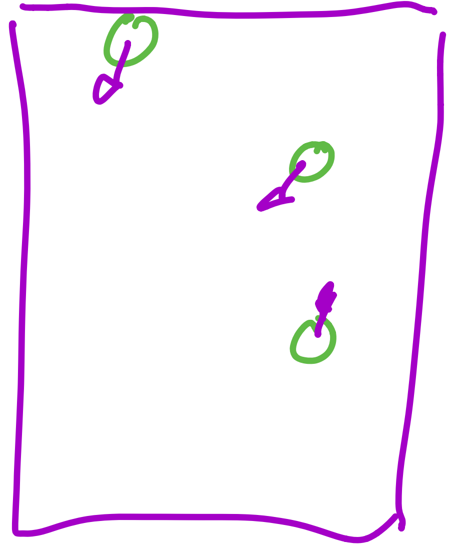
1 pose, 3 points

$$\min_{\substack{\Delta T \\ \Delta p}} \sum_j \left\| h_0({}_wT_i^0, p_w^{j0}) + H\Delta T + G\Delta p - z_{ij} \right\|^2$$



UPDATE





VO pipeline.